

Sandage-Loeb test for the new agegraphic and Ricci dark energy models

Jingfei Zhang, Li Zhang, and Xin Zhang

Department of Physics, College of Sciences, Northeastern University, Shenyang 110004, China

The Sandage-Loeb (SL) test is a unique method to explore dark energy at the “redshift desert” ($2 \lesssim z \lesssim 5$), an era not covered by any other dark energy probes, by directly measuring the temporal variation of the redshift of quasar (QSO) Lyman- α absorption lines. In this paper, we study the prospects for constraining the new agegraphic dark energy (NADE) model and the Ricci dark energy (RDE) model with the SL test. We show that, assuming only a ten-year survey, the SL test can constrain these two models with high significance.

I. INTRODUCTION

The astronomical observations of type Ia supernovae (SNIa) indicate that our universe is undergoing an accelerating expansion [1]. This cosmic acceleration has also been confirmed by other observations, such as the large scale structure (LSS) [2] and the cosmic microwave background (CMB) [3]. Nowadays it is the most accepted idea that a mysterious dominant component, dark energy, with large enough negative pressure, is responsible for this cosmic acceleration. Among all theoretical models, the preferred one is the so-called Λ CDM model, which consists of a mixture of Einstein’s cosmological constant Λ and the cold dark matter (CDM). The Λ CDM model provides an excellent explanation for the acceleration of the universe and the existing observational data. However, the cosmological constant has to face severe theoretical problems such as the puzzle why the dark energy density today is so small compared to typical particle scales. Therefore, except the Λ CDM model, many dynamical dark energy models have been proposed, in which the equation of state (EoS) of dark energy is no longer a constant but slightly evolves with time. For reviews of dark energy, see, e.g., Ref. [4].

In the face of so many candidate models, it is extremely important to identify which one is the correct model by using the observational data. The measurement of the expansion rate of the universe at different redshifts is crucial to discriminate these competing candidate models. Up to now, a number of cosmological tools have been used to successfully probe the expansion and the geometry of the universe. These, typically, include the luminosity distance of SNIa, the position of acoustic peaks in the CMB angular power spectrum, and the scale of baryon acoustic oscillations (BAO) in the power spectrum of matter extracted from galaxy catalogues. Recently, the application of time evolution of cosmological redshift as a test of dark energy models has become attractive, since this method opens a new window of exploring the “redshift desert” ($2 \lesssim z \lesssim 5$). In addition to being a direct probe of the dynamics of the expansion, this method has the

advantage of not relying on a determination of the absolute luminosity of the observed sources, but only on the identification of stable spectral lines, so this method can reduce the uncertainties from systematic or evolutionary effects.

Sandage [5] was the first to propose the possible application of this kind of observation as a cosmological tool. However, owing to the tininess of the expected variation, this observation was deemed impossible at that time. In 1998, Loeb [6] revisited this suggestion and argued that the redshift variation of quasar (QSO) Lyman- α absorption lines could be detected in the not too distant future, given the advancement in technology occurred over the last forty years. In fact, the cosmological redshift variation at 1σ would be detected in a few decades, if a sample of a few hundred QSOs could be observed with high resolution spectroscopy with a ten meter telescope. This method is usually referred to as “Sandage-Loeb” (SL) test. The possibility of detecting the temporal variation of redshift with the Cosmic Dynamics Experiment (CODEX) was first analyzed by Corasaniti, Huterer and Melchiorri [7]. Their work [7] has provided the first quantitative analysis of the SL test, from which all other analyses have followed.

In Ref. [7], Corasaniti, Huterer and Melchiorri employed the SL test to constrain dark energy models such as Λ CDM model, Chaplygin gas model, and interacting dark energy model. Later, Balbi and Quercellini [8] extended this analysis to more dark energy models including constant EoS model, variable EoS model, interacting dark energy model, DGP model, Cardassian model, generalized Chaplygin gas model, affine EoS model, etc. More recently, Zhang, Zhong, Zhu and He [9] further used the SL test to explore the holographic dark energy model. However, it should be pointed out that there are three holographic dark energy models: the original holographic dark energy model [10], the new agegraphic dark energy model [11], and the Ricci dark energy model [12]. Actually, in Ref. [9], only the original holographic dark energy model [10], i.e., the model in which the IR cutoff is given by the future event horizon, was investigated. Thus, along this line, as a next step, one should further explore the new agegraphic and the Ricci dark energy models with the SL test. In this paper, this will be done. This will provide a complementary to the work of Ref. [9] and keep the investigation of holographic dark energy models more complete.

In the subsequent section, we will briefly review the new agegraphic dark energy model and the Ricci dark energy model. In Sec. III, we will explore these two models with the SL test. In the last section, we will give some concluding remarks.

II. NEW AGEGRAPHIC AND RICCI DARK ENERGY MODELS

In this section, we will briefly review the new agegraphic dark energy model and the Ricci dark energy model. In fact, these two models both belong to the holographic scenario of dark energy.

It is well known that the holographic principle is an important result of the recent research for exploring the quantum gravity [13]. This principle is enlightened by investigations of the quantum property of black holes. In a quantum gravity system, the conventional local quantum field theory will break down because it contains too many degrees of freedom that would lead to the formation of a black hole breaking the effectiveness of the quantum field theory. To reconcile this breakdown with the success of local quantum field theory in describing observed particle phenomenology, some authors proposed a relationship between the ultraviolet (UV) and the infrared (IR) cutoffs due to the limit set by the formation of a black hole [14]. The UV-IR relation in turn provides an upper bound on the zero-point energy density. In other words, if the quantum zero-point energy density ρ_{vac} is relevant to a UV cutoff, the total energy of the whole system with size L should not exceed the mass of a black hole of the same size, and thus we have $L^3 \rho_{\text{vac}} \leq LM_{\text{Pl}}^2$. This means that the maximum entropy is of the order of $S_{\text{BH}}^{3/4}$. When we take the whole universe into account, the vacuum energy related to this holographic principle is viewed as dark energy, usually dubbed holographic dark energy (its density is denoted as ρ_{de} hereafter).

The largest IR cutoff L is chosen by saturating the inequality so that we get the holographic dark energy density [10]

$$\rho_{\text{de}} = 3c^2 M_{\text{Pl}}^2 L^{-2}, \quad (1)$$

where c is a numerical constant, and $M_{\text{Pl}} \equiv 1/\sqrt{8\pi G}$ is the reduced Planck mass. If we take L as the size of the current universe, for instance the Hubble radius H^{-1} , then the dark energy density will be close to the observational result. However, if one takes the Hubble scale as the IR cutoff, the holographic dark energy seems not to be capable of leading to an accelerating universe [15]. The first viable version of holographic dark energy model was proposed by Li [10]. In this model, the IR length scale is taken as the event horizon of the universe. The holographic dark energy model based on the event horizon as the IR cutoff has been widely studied [16] and found to be consistent with the observational data [17, 18].

There are also other two versions of the holographic dark energy, i.e., the new agegraphic dark energy model [11, 19] and the Ricci dark energy model [12, 20, 21]. For the new agegraphic dark energy model, the IR scale cutoff is chosen to be the conformal age of the universe; for the Ricci dark energy model, the IR cutoff is taken as the average radius of the Ricci scalar curvature. We shall briefly review these two models in the following subsections.

A. New agegraphic dark energy model

For a spatially flat (the assumption of flatness is motivated by the inflation scenario) Friedmann-Robertson-Walker (FRW) universe with matter component ρ_m and dark energy component ρ_{de} , the Friedmann equation reads

$$3M_{\text{Pl}}^2 H^2 = \rho_m + \rho_{de} , \quad (2)$$

or equivalently,

$$E(z) \equiv \frac{H(z)}{H_0} = \left(\frac{\Omega_{m0}(1+z)^3}{1 - \Omega_{de}} \right)^{1/2} , \quad (3)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter, Ω_{m0} is the present fractional matter density, and $\Omega_{de} \equiv \frac{\rho_{de}}{\rho_c} = \frac{\rho_{de}}{3M_{\text{Pl}}^2 H^2}$ is the fractional dark energy density.

In the old version of the agegraphic dark energy model [22], the IR cutoff is chosen as the age of the universe T (here it should be pointed out that the light speed has already been taken as 1, so time and length have the same dimension). However, there are some inner inconsistencies in this model; for details see Ref. [11]. So, in this paper, we only discuss the new version of the agegraphic dark energy model. In the new agegraphic dark energy model, the IR cutoff is chosen to be the conformal age of the universe,

$$\eta \equiv \int_0^t \frac{dt}{a} = \int_0^a \frac{da}{a^2 H} , \quad (4)$$

so the density of the new agegraphic dark energy is

$$\rho_{de} = 3n^2 M_{\text{Pl}}^2 \eta^{-2} . \quad (5)$$

To distinguish from the original holographic dark energy model, a new constant parameter n is used to replace the former parameter c . Taking derivative for Eq. (5) with respect to $x = \ln a$ and making use of Eq. (4), we get

$$\rho'_{de} = -2\rho_{de} \frac{\sqrt{\Omega_{de}}}{na} . \quad (6)$$

This means that the EoS of the new agegraphic dark energy is

$$w_{de} = -1 + \frac{2}{3n} \frac{\sqrt{\Omega_{de}}}{a} . \quad (7)$$

Taking derivative for $\Omega_{de} = n^2/(H^2 \eta^2)$, and considering Eq. (4), we obtain

$$\Omega'_{de} = 2\Omega_{de} \left(\epsilon - \frac{\sqrt{\Omega_{de}}}{na} \right) , \quad (8)$$

where

$$\epsilon = \frac{3}{2}(1 + w_{\text{de}}\Omega_{\text{de}}) = \frac{3}{2} - \frac{3}{2}\Omega_{\text{de}} + \frac{\Omega_{\text{de}}^{3/2}}{na}. \quad (9)$$

Hence, we get the equation of motion for Ω_{de} ,

$$\Omega'_{\text{de}} = \Omega_{\text{de}}(1 - \Omega_{\text{de}}) \left(3 - \frac{2}{n} \frac{\sqrt{\Omega_{\text{de}}}}{a} \right), \quad (10)$$

and this equation can be rewritten as

$$\frac{d\Omega_{\text{de}}}{dz} = -\Omega_{\text{de}}(1 - \Omega_{\text{de}}) \left(3(1+z)^{-1} - \frac{2}{n} \sqrt{\Omega_{\text{de}}} \right). \quad (11)$$

As in Ref. [11], we choose the initial condition, $\Omega_{\text{de}}(z_{\text{ini}}) = n^2(1+z_{\text{ini}})^{-2}/4$, at $z_{\text{ini}} = 2000$, then Eq. (11) can be numerically solved. Substituting the results of Eq. (11) into Eq. (3), the function $E(z)$ can be obtained. Notice that once n is given, $\Omega_{\text{m}0} = 1 - \Omega_{\text{de}}(z=0)$ can be naturally obtained by solving Eq.(11), so the new agegraphic dark energy model is a single-parameter model.

B. Ricci dark energy model

For a spatially flat FRW universe, the Ricci scalar is

$$\mathcal{R} = -6(\dot{H} + 2H^2). \quad (12)$$

As suggested by Gao et al. [12], the energy density of Ricci dark energy is

$$\rho_{\text{de}} = \frac{3\alpha}{8\pi G} (\dot{H} + 2H^2) = -\frac{\alpha}{16\pi G} \mathcal{R}, \quad (13)$$

where α is a positive numerical constant to be determined by observations. Comparing to Eq. (1), it is seen that if we identify the IR cutoff L^{-2} with $-\mathcal{R}/6$, we have $\alpha = c^2$. As pointed out by Cai et al. [23], the Ricci dark energy can be viewed as originated from taking the causal connection scale as the IR cutoff in the holographic setting. Now, the Friedmann equation, in a universe containing Ricci dark energy and matter, can be written as

$$H^2 = \frac{8\pi G}{3} \rho_{\text{m}0} e^{-3x} + \alpha \left(\frac{1}{2} \frac{dH^2}{dx} + 2H^2 \right), \quad (14)$$

and this equation can be further rewritten as

$$E^2 = \Omega_{\text{m}0} e^{-3x} + \alpha \left(\frac{1}{2} \frac{dE^2}{dx} + 2E^2 \right), \quad (15)$$

where $E \equiv H/H_0$. Solving this equation, and using the initial condition $E_0 = E(t_0) = 1$, we have

$$E(z) = \left(\frac{2\Omega_{\text{m}0}}{2-\alpha} (1+z)^3 + \left(1 - \frac{2\Omega_{\text{m}0}}{2-\alpha} \right) (1+z)^{(4-\frac{2}{\alpha})} \right)^{1/2}. \quad (16)$$

There are two model parameters, $\Omega_{\text{m}0}$ and α , in the Ricci dark energy model.

III. THE SANDAGE-LOEB TEST

In this section, we will first review the Sandage-Loeb test, and then explore the new agegraphic dark energy model and the Ricci dark energy model with the SL test.

First, let us consider an isotropic source emitting at rest. The well-known redshift relation of the radiation emitted by the source at t_s and observed at t_o is

$$z_s(t_o) = \frac{a(t_o)}{a(t_s)} - 1. \quad (17)$$

Furthermore, consider lights emitted after a period Δt_s at $t_s + \Delta t_s$ and detected later at $t_o + \Delta t_o$. Obviously, the observed redshift of the source at $t_o + \Delta t_o$ is

$$z_s(t_o + \Delta t_o) = \frac{a(t_o + \Delta t_o)}{a(t_s + \Delta t_s)} - 1. \quad (18)$$

Therefore, the variation of the source redshift between times t_o and $t_o + \Delta t_o$ would be measured as follows:

$$\Delta z_s \equiv \frac{a(t_o + \Delta t_o)}{a(t_s + \Delta t_s)} - \frac{a(t_o)}{a(t_s)}. \quad (19)$$

We can expand the ratio $a(t_o + \Delta t_o)/a(t_s + \Delta t_s)$ to linear order, under the approximation $\Delta t/t \ll 1$. Furthermore, using the relation $\Delta t_o = [a(t_o)/a(t_s)]\Delta t_s$, we obtain

$$\Delta z_s \approx \left[\frac{\dot{a}(t_o) - \dot{a}(t_s)}{a(t_s)} \right] \Delta t_o. \quad (20)$$

It shows that the redshift variation Δz_s is directly related to a change in the expansion rate during the evolution of the universe, and it is thus a direct probe of the dynamics of the cosmic expansion. This redshift variation can be related to a spectroscopic velocity shift, $\Delta v \equiv \Delta z_s/(1 + z_s)$. Using the Hubble parameter $H(z) = \dot{a}(z)/a(z)$, we obtain

$$\Delta v = H_0 \Delta t_o \left[1 - \frac{E(z_s)}{1 + z_s} \right], \quad (21)$$

where H_0 is the Hubble constant and $E(z) = H(z)/H_0$. The function $E(z)$ contains all the details of the cosmological model under investigation. It is clear that the expansion history $E(z)$ is related to the spectroscopic velocity shift via Eq. (21).

Though the amplitude of the velocity shift is very small, the absorption lines in the quasar Lyman- α provide us with a powerful tool to detect such a small signal. Monte Carlo simulations of Lyman- α absorption lines have been performed to estimate the uncertainty on Δv as measured by the CODEX spectrograph [24]. The statistical error can be estimated as

$$\sigma_{\Delta v} = 1.4 \left(\frac{2350}{S/N} \right) \sqrt{\frac{30}{N_{QSO}}} \left(\frac{5}{1 + z_{QSO}} \right)^{1.8} \frac{\text{cm}}{\text{s}}, \quad (22)$$

where S/N denotes the spectral signal-to-noise defined per 0.0125 \AA pixel, N_{QSO} is the number of Lyman- α quasars, and z_{QSO} is the quasar's redshift. In order to detect the cosmic signal, a large S/N is necessary, but this implies that a positive detection is not feasible with current telescopes. Fortunately, the CODEX under design will be installed on the ESO Extremely Large Telescope. The necessary signal-to-noise can be reached by such an about 50 meter giant with just few hours integration. The velocity shift measurements open a cosmological window with particular focus on dark energy models. From the velocity shift measurements, one can forecast constraints on parameters of cosmological models. In this paper, following Refs. [7–9], we consider experimental configuration and uncertainties similar to those expected from CODEX. We assume that the survey would observe a total of 240 QSOs uniformly distributed in six equally spaced redshift bins in the range $2 \lesssim z \lesssim 5$, with a signal-to-noise $S/N = 3000$, and the expected uncertainty as given by Eq. (22). Also, in this paper, we consider a ten-year survey, namely, $\Delta t_o = 10$ years. In what follows, we shall explore the new agegraphic dark energy model and the Ricci dark energy model with the SL test.

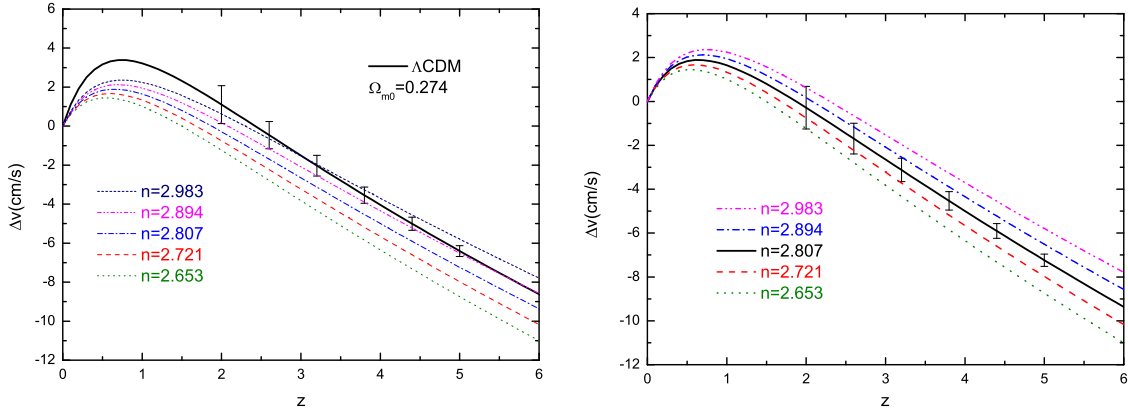


FIG. 1: The SL test for the NADE model. In the left panel, the Λ CDM model with $\Omega_{m0} = 0.274$ is used to be the fiducial model; in the right panel, the NADE model with $n = 2.807$ is taken as the fiducial model.

First, we discuss the velocity shift behavior in the new agegraphic dark energy (NADE) model. Numerically solving the differential equation (11) and inserting Eq. (3) into Eq. (21), one can reconstruct the $\Delta v(z)$ curves for the NADE model. Note that the NADE model is a single-parameter model, and the sole model parameter is n . The current cosmological constraint on the parameter n is $n = 2.807^{+0.087}_{-0.086} (1\sigma) {}^{+0.176}_{-0.170} (2\sigma)$ [17]. In this work, we take the values of n as the central value as well as the one-sigma and two-sigma limits of the above observational constraint. In Fig. 1, we reconstruct the velocity shift behavior of the NADE model. In the left panel, we use the Λ CDM model as a fiducial model to perform an SL test. For the Λ CDM cosmology, Ω_{m0} is chosen to be 0.274 as given by the WMAP five-year observations [25]. Note

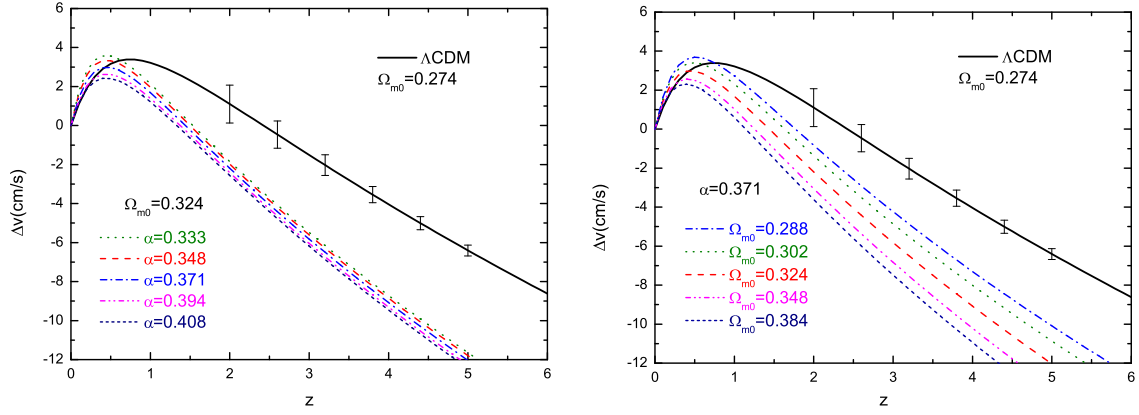


FIG. 2: The SL test for the RDE model. The Λ CDM model with $\Omega_{m0} = 0.274$ is used to be the fiducial model. In the left panel, we fix $\Omega_{m0} = 0.324$ and vary α ; in the right panel, we fix $\alpha = 0.371$ and vary Ω_{m0} .

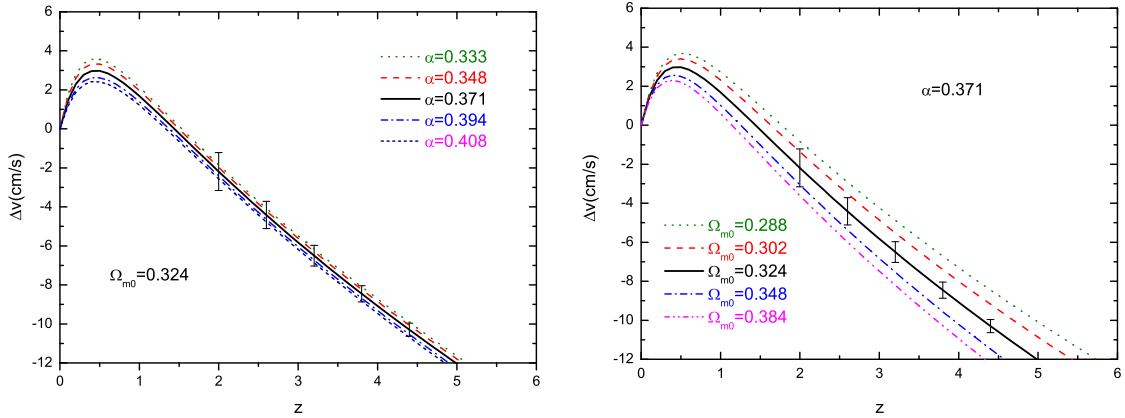


FIG. 3: The SL test for the RDE model. The RDE model with $\Omega_{m0} = 0.324$ and $\alpha = 0.371$ is used to be the fiducial model. In the left panel, we fix $\Omega_{m0} = 0.324$ and vary α ; in the right panel, we fix $\alpha = 0.371$ and vary Ω_{m0} .

also that the Hubble constant H_0 in Eq. (21) is taken as $72 \text{ km s}^{-1} \text{ Mpc}^{-1}$, in the whole discussion. From the left panel of Fig. 1 we see that the NADE model can be distinguished from the Λ CDM model via the SL test. However, it is interesting to notice that the curve with $n = 0.2894$ still lies in the 1σ error range of the Λ CDM fiducial model within the redshift desert $2 \lesssim z \lesssim 5$. So, it is fair to say that actually the SL test could not completely distinguish the NADE model from the Λ CDM model, though it can do it rather effectively. Of course, if the SL test is combined with the low-redshift observations such as the SNIa, weak lensing, and BAO, the NADE model can be completely distinguished from the Λ CDM model. On the other hand, we also perform an SL test by using the NADE model with $n = 2.807$ (the central value of n given by the current observational constraint) as a fiducial model. We show this case in the right panel of Fig. 1. In this case, we see that the prospective SL test is very powerful to be used to constrain the NADE model, and

it is clearly better than the current low-redshift observations.

Next, we switch to the Ricci dark energy (RDE) model. For the RDE model, the Hubble expansion history is described by Eq. (16), so the $\Delta v(z)$ behavior can be reconstructed by substituting Eq. (16) into Eq. (21). The RDE model has two model parameters, α and Ω_{m0} , and the current observational constraint results are [17]: $\alpha = 0.371^{+0.023}_{-0.023}$ (1σ) $^{+0.037}_{-0.038}$ (2σ) and $\Omega_{m0} = 0.324^{+0.024}_{-0.022}$ (1σ) $^{+0.040}_{-0.036}$ (2σ). Like the above discussion, we will still first employ the standard dark energy cosmology, the Λ CDM model, as a fiducial model to perform an SL test for the RDE model. For the fiducial Λ CDM model, we still take $\Omega_{m0} = 0.274$, the value given by WMAP [25]. In this test, one can forecast how well a deviation of the RDE model from the Λ CDM model can be detected. We show the results of such an SL test in Fig. 2. In the left panel, the parameter Ω_{m0} is fixed to be 0.324 for the RDE model, and the parameter α of the RDE model is adjustable; in the right panel, we fix $\alpha = 0.371$ and vary Ω_{m0} for the RDE model. From this figure, it is clear to see that the SL test in the “redshift desert” is very successful in distinguishing the RDE model from the standard Λ CDM cosmology. The two models can be distinguished via the SL test at about $6 - 8 \sigma$ level, assuming only a 10-year survey. This result is reasonable and understandable, since the RDE model has a tracker behavior [20] so that its matter-dominated era is different from that of the Λ CDM model, and the SL test is just the best way to probe the behavior of the matter-dominated phase of dark energy models. Furthermore, it is worth noticing that such measurements are mostly sensitive to the matter density Ω_{m0} , while the dependence on α is weaker. To forecast whether the SL test is able to break the degeneracy of the parameters of the RDE model, we further use the RDE model with $\Omega_{m0} = 0.324$ and $\alpha = 0.371$ (the central values of the current observational constraint) as the fiducial model to perform an SL test. The results of this test are plotted in Fig. 3. In the left panel, we fix $\Omega_{m0} = 0.324$ and vary α in the 2σ range of the current observational result; in the right panel, we fix $\alpha = 0.371$ and vary Ω_{m0} in the 2σ range of the current observational result. From Fig. 3, we can see that the SL test with a 10-year survey could not provide precision determination of the parameter α , but could precisely determine the value of Ω_{m0} for the RDE model. This result is in agreement with the previous work on the SL test [7]; in Ref. [7] it is shown that the Sandage-Loeb constraints on w are not competitive with those inferred from other low-redshift observations. The equation of state w of RDE is mainly determined by the parameter α , so the SL test could not constrain the value of α precisely. However, as pointed out in Ref. [7], one should note that the constraints obtained by SL test decrease linearly with time, so for measurements made over a century, and with the expected larger number of QSOs, the SL limits on w can easily be at the few percent level.

Finally, let us show how well the proposed SL test could constrain the two dark energy models by using a Fisher matrix method. To find the expected precision of the SL test with CODEX, one must assume a fiducial model, and then simulate the experiment assuming it as a reference model. We employ the Λ CDM

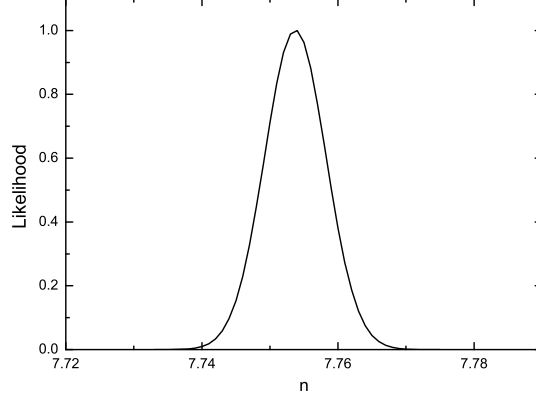


FIG. 4: Predicted likelihood distribution of the parameter n of the NADE model from the SL test.

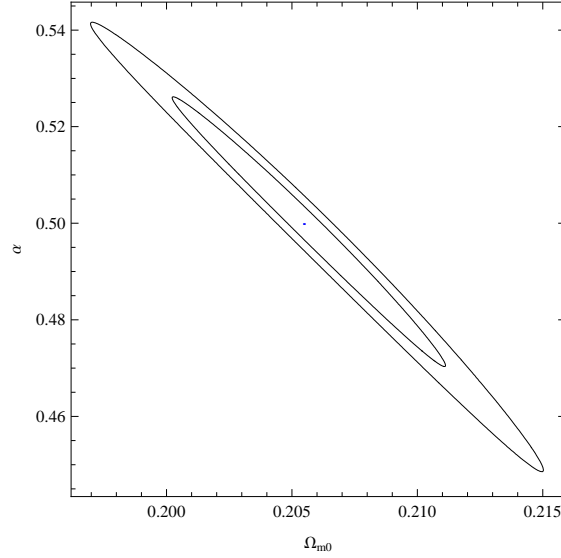


FIG. 5: Predicted probability contours at 68.3% and 95.4% confidence levels in the (Ω_{m0}, α) plane for the RDE model from the SL test.

model as the fiducial model and produce the mock data of the velocity-drift in the redshift desert with the error bars given by Eq. (22). Next, we will perform a Fisher matrix analysis on the model parameter space with the above assumption. The χ^2 of the analysis is given by

$$\chi_{SL}^2 = \sum_{i=1}^{240} \frac{[\Delta v^{\text{model}}(z_i) - \Delta v^{\text{data}}(z_i)]^2}{\sigma_{\Delta v}^2(z_i)}, \quad (23)$$

where $\Delta v^{\text{data}}(z_i)$ denotes the mock data produced by the fiducial model, $\Delta v^{\text{model}}(z_i)$ is the theoretical prediction of the dark energy model under investigation, and $\sigma_{\Delta v}^2(z_i)$ is the error bar estimated from Eq. (22). The NADE model is a single-parameter model, so once the parameter n is given, the parameter Ω_{m0} as well as

other properties and dynamical evolution will be determined accordingly. In Fig. 4 we plot the likelihood distribution of the parameter n of the NADE model as expected from the SL test. We see that the parameter n is able to be accurately determined by the SL test, $n = 7.754^{+0.004}_{-0.005}$ (1σ). So, the SL test could obtain $\sigma_n \simeq 0.005$, better than the current observational result by at least one order of magnitude. The RDE model has two model parameters, Ω_{m0} and α . In Fig. 5 we plot the 1σ and 2σ contours in the $\Omega_{m0} - \alpha$ plane for the RDE model. The 1σ results are: $\Omega_{m0} = 0.2055^{+0.0056}_{-0.0052}$ and $\alpha = 0.5000^{+0.0262}_{-0.0297}$. So, the SL test would obtain $\sigma_{\Omega_{m0}} \simeq 0.005$, better than the current constraint by one order of magnitude, and $\sigma_\alpha \simeq 0.03$, at the same accuracy level compared to the current observational result. The above analysis reinforces the conclusion that the two dark energy models indeed can be constrained by the SL measurements with high significance.

IV. CONCLUDING REMARKS

The Sandage-Loeb test is a promising method for constraining dark energy by using the direct measurements of the temporal shift of the quasar Lyman- α absorption lines at high redshift ($2 \lesssim z \lesssim 5$). While the signal of this effect is extremely small, the near-future large telescopes with ultrahigh resolution spectrographs (such as the CODEX under design) will definitely be capable of measuring such a small signal over a period as short as ten years. Notwithstanding, one still may ask whether it is worthwhile to probe $z \gtrsim 2$, since in the standard dark energy cosmology such as the Λ CDM model or the slowly rolling scalar field model the dark energy is subdominant at redshift $z \gtrsim 1$ and almost completely negligible at $z \gtrsim 2$. The answer to this question is affirmative. As explained in Ref. [7], it is quite rational to look for the signatures of dark energy at all available epochs, because at present we do not know much about the physical nature and cosmological origin of dark energy, and there are too many possibilities for dark energy. Especially, there are lots of dark energy models in which dark energy density is non-negligible at high redshift. So, it is meaningful to study the future observations of velocity shift and their impact on dark energy models. The SL test on Chaplygin gas model and interacting dark energy model has been studied in Ref. [7]. In Ref. [8], more classes of dark energy models have been investigated with the SL test. In particular, the original holographic dark energy model has been explored with the SL test in Ref. [9]. So, the exploration of the other two holographic dark energy models, the new agegraphic dark energy model and the Ricci dark energy model, is naturally the next step, and this will provide a complementary to the work of Ref. [9] and keep the investigation of holographic scenarios of dark energy more complete.

In this paper, we have analyzed the prospects for constraining the new agegraphic dark energy model and the Ricci dark energy model at the redshift desert $2 \lesssim z \lesssim 5$ from the Sandage-Loeb test. The NADE model is a sole-parameter model; the parameter n together with an initial condition can determine the whole

cosmological evolution history of the NADE model. Actually, the evolution of this model is similar to that of a slowly rolling scalar field. Thus, though the SL test can be used to distinguish the NADE model from the Λ CDM model, this discrimination is not absolute. It is obvious that if the SL test is combined with the low-redshift observations such as the SNIa, weak lensing, and BAO, the NADE model can be completely distinguished from the Λ CDM model. Furthermore, we forecast that the prospective SL test is very powerful to constrain the NADE model, and it is better than the current low-redshift observations. The RDE model is a two-parameter model, and it has a tracking solution so that it is very different from the Λ CDM model in the matter-dominated epoch. So, the SL test is very successful in distinguishing the RDE model from the Λ CDM model. The two models can be distinguished via the SL test at about $6 - 8 \sigma$ level, assuming only a 10-year survey. As the SL test mostly probes the matter density at high redshift, the constraint on Ω_{m0} is very strong, while the constraint on α is much weaker.

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